Optimal Task Design for Intrinsically Motivated Workers with an Incomplete Contract

Keiki Kumagae*

This paper presents analysis of the optimal task design problem for intrinsically motivated workers within a firm with an incomplete contract theory. The authors study the control problem in the context of a search for projects and the interplay between optimal task design and intrinsic motivation. Our model is based on that reported by Bao and Wang (Journal of Economics 107 81-96, 2012), where optimal organization structure is discussed. Bao and Wang (2012) report that, because workers exert maximum effort to discover a project’s information to implement their favorite project, firms invariably choose integration without bargaining. The main result of this paper is the observation that when intrinsic motivation is sufficiently high, the firm might choose separation, in contrast to results reported by Bao and Wang. The higher a worker’s intrinsic motivation is, the higher the probability of choosing the firm management’s favorite project becomes. Consequently, although intrinsic motivation increases the firm’s profit, it decreases the worker’s incentive to exert effort to be informed.

[Keywords : Task design, Incomplete contract, Intrinsic motivation]

1. Introduction

In Human Resource Management, it is important for firm managers to consider not only extrinsic motivation of workers such as rewards but also their intrinsic motivation. Most firm managers actively give information about the firm...
management strategy and goals, periodically conduct interviews with workers and set specific targets for achievements to induce intrinsic motivation from workers.

The task design problem in a firm has been discussed in economics. Lewis and Sappington (1997) and Khalil, Kim and Shin (2006) examine optimal task design to assign planning and implementation tasks to agents. However, no report of the relevant literature describes exploration of how worker’s intrinsic motivation affects optimal task design.

As described in this paper, we analyze an optimal task design problem for intrinsically motivated workers within a firm. Whereas a firm usually assigns tasks to workers considering the scope of authority, characteristics of their tasks, and personalities of workers, task design also changes the intrinsic motivation of workers. The worker’s level of self-determination increases if much authority of tasks is assigned to one worker. Increasing the level of self-determination improves job satisfaction. The worker is intrinsically motivated for a task. Consequently, workers having a feeling of high satisfaction work for that firm more altruistically.

Our model is based on that announced by Bao and Wang (2012), who study the control problem in the context of a search for projects. The firm management must decide on the allocation of control of two divisions to two workers. The firm management decides whether to have two separate divisions and give each worker some authority (separation), or to integrate the divisions with only one worker (integration). Each worker searches for information related to projects. Workers with authority can decide which project is implemented. In each division, when one agent is informed and the other is not, the agent being informed must agree to the informed agent’s preferred project. The agent having authority can choose a favorite one if both agents have information. Bao and Wang (2012) particularly demonstrate that integration is always chosen without bargaining between agents because workers exert maximum effort to seek out the project’s information to implement a favorite one in two divisions.

Our work differs from works reported in the relevant literature because our work rests on the assumption that the worker is intrinsically motivated by task design and that the worker comes to exhibit altruistic behavior. A worker with integrated authority has intrinsic motivation because of high levels of self-determination. Another necessary assumption is that the motivated worker chooses not only the worker’s favorite project but also the firm management’s favorite with a probability indicating the extent of the intrinsic motivation.

Furthermore, we introduce the worker’s screening ability for correct evaluation of the project under integration. The worker in control has a superior role to that of the worker losing control under integration. The superior worker must evaluate the project proposed by the subordinate worker. For this study, another assumption is that the superior worker might erroneously reject the project offer. The model of the screening ability relies on some observations of De
Paola and Scoppa (2006), who conducted a comparison between hierarchical and decentralized organizations in agency theory, and who assumed that projects proposed by the agent are evaluated by the principal in a hierarchical organization. The principal makes erroneous judgments and sometimes rejects profitable proposals or approves bad ones.

The main results presented herein are that when intrinsic motivation is high and screening ability is low, a firm manager might choose separation, in contrast to results reported by Bao and Wang (2012). The higher a worker’s intrinsic motivation is, the higher the probability of choosing the firm management’s favorite project becomes. Consequently, although intrinsic motivation increases the firm’s profit, it decreases the worker’s effort incentive to search. Integration presents the important benefit of inducing maximum effort, as described by Bao and Wang (2012), but such a benefit might be lost in our model.

The result presented herein suggests implications for optimal task design in a firm. Firms usually want to assign many integrated tasks to the intrinsically motivated worker because he works for the firm’s profit. However, once the worker notices that using altruistic behavior yields less benefit, the worker might lose incentive to greater effort. Therefore, integration is not always optimal for a firm.

Our work is related to studies reported by Makris (2009) and Delfgaauw and Dur (2008). They too assume that the worker is intrinsically motivated and study interaction of the optimal contracts and the extent of intrinsic motivation with a complete contract. As described by Makris (2009), an agent having warm-glow altruism emotion derives utility from part of the principal’s profit. In a report of a study by Delfgaauw and Dur (2008), the agent feels pleasure in his personal contribution to the firm’s goal. Therefore, he derives intrinsic utility from exertion of effort. In our model, the agent does not derive utility from intrinsic motivation, but increases altruistic behavior for the firm to model the relation between the pattern of task design and intrinsic motivation.

The work described herein is also related to research reported by Lewis and Sappington (1997): the principal assigns two tasks (acquiring valuable planning information and reducing the cost of projects) to agents in the task design problem literature. They show that it is optimal for the principal to separate the planning and implementation tasks by assigning these tasks to two agents because the principal can motivate the agent to exert effort to acquire and report the planning information truthfully without information rent by separating the tasks. Lewis and Sappington (1997) rely on the assumption that the principal can use complete contracts based on the cost of projects because the cost is observable and verifiable. As described in this paper, neither the effort exerted by agents nor the payoffs from implementing projects are verifiable. For that reason, the contract is incomplete.

The organization of the paper is the following. Section 2 presents the model. Section 3 discusses benchmark cases, whereas Section 4 solves the levels of effort in equilibrium and finds the optimal task design. Section 5 concludes this paper.
2. The Model

This study is based on the model presented by Bao and Wang (2012). A principal must hire agents to implement a project. We assume that all parties are risk neutral. The principal deals with two agents: $A_1$ and $A_2$. The principal does not seek information of projects. Each agent seeks the information. Two divisions $(D_1, D_2)$ each have $N \geq 3$ projects. Each project $n \in \{1, 2, 3, \ldots, N\}$ is associated with a profit $B_n$ to the principal and a private benefit $b_n^i$ to $A_i$ ($i = 1, 2$). For each party, at least one project’s payoff is sufficiently negative. Consequently, when agents are uninformed about the project’s information, no project is implemented. No project yields 0 payoffs to each party. Each player can receive a payoff from both $D_1$ and $D_2$.

At private cost $g(e)$, he can perfectly obtain information about the payoffs with probability $e_i$. We assume that $g(e)$ is increasing and strictly convex ($g'(e) > 0$, $g''(e) > 0$). Let $e_i$ denote the effort selected by $A_i$.

The principal’s favorite project, $F_p$, yields profit $B$ to the principal, $\beta b$ to agents. The agent $A_i$’s favorite project, $F_i$, yields profit $\alpha B$ to the principal, $b$ to $A_i$, and $\gamma b$ to $A_j$ ($i, j \in \{1, 2\}, i \neq j$). We assume that $\alpha, \beta, \gamma$ are of $(0,1)$. All parameters are common knowledge. We also assume that two or more projects yield equal payoffs. For simplicity $B + \beta b + \beta b > \alpha B + b + \gamma b$ is assumed, which implies that the joint payoff for all three parties cannot be improved when $F_p$ is implemented. Information related to the project’s payoff is soft information. Therefore, the informed agents cannot report their information.

The principal decides to allocate the authority to choose the project implemented in each division to agents. If the principal gives one agent authority for $D_1$ and another authority for $D_2$, then we say that the principal has chosen task separation. If the principal gives the same agent to the authority for both divisions, then the principal has chosen task integration (presented in Fig. 1). In each division, when one agent has information and the other does not, the agent with no information must agree to the informed agent’s preferred project. When both agents are informed, the agent having the authority can choose a favorite project.

![Fig. 1 : Pattern of Task Design](image-url)
When the principal chooses integration, $A_1$ can use the discretion of the projects in two divisions. Furthermore, $A_1$ has responsibility for the firm or a feeling of satisfaction. Then, in our model, the agent is intrinsically motivated and displays altruistic behavior toward the firm. The informed agent then chooses $F_p$ with probability $\theta \in (0, 1)$ and $F_1$ with probability $1 - \theta$. Parameter $\theta$ represents the extent of the agent’s intrinsic motivation. Assume that $\theta$ is common knowledge.

By contrast, under integration, $A_1$ is a superior worker for $A_2$. $A_1$ evaluates the proposed project made by $A_2$. Assuming that $t$ denotes the screening ability of $A_1$, Then, $A_1$ apprrove the proposal $(F_p)$ made by $A_2$ with probability $t$, whereas the worker erroneously rejects $F_p$ with probability $1 - t$. For simplicity, we assume that $1 - \theta(1 - \beta) - t\gamma \geq 0$, which implies that the optimal levels of effort are the interior solution in section 4.1. We summarize the structure of this game in Fig. 2. The three values in parentheses in this figure are, respectively, the firm’s payoff, $A_1$’s payoff, and $A_2$’s payoff when $F_k$ or no project is implemented ($k \in \{1, 2, P\}$). Figure 2 is not the game tree.

3. Benchmark Case without Asymmetric Information

As explained in this section, the principal can design contracts to ensure that her preferred project is implemented for both divisions when at least one agent obtains information with probability $1 - (1 - e_1)(1 - e_2) = e_1 + e_2 - e_1 e_2$. In this case, no task design problem arises. The expected social welfare is

$$2(e_1 + e_2 - e_1 e_2)(B + \beta b + \beta b) - g(e_1) - g(e_2).$$
Let $e_1^{FB}, e_2^{FB}$ denote effort levels when the expected social welfare is maximized. We index the solution to this problem with a superscript FB meaning “first-best”. The first-order condition is

$$g'(e_1^{FB}) = 2(1 - e_2^{FB})(B + \beta b + \beta b), \quad \ldots(1)$$

$$g'(e_2^{FB}) = 2(1 - e_1^{FB})(B + \beta b + \beta b), \quad \ldots(2)$$

For simple calculation, it is apparent that their efforts are strategic substitutes. Because $F_P$ is chosen by the principal when at least one agent is informed, the efforts of two agents are substitutes to save the sum of effort’s costs.

4. Optimal Task Design

This section presents consideration of the case in which the information of projects is not verifiable and the contracts are incomplete. Letting $\prod_k^{SP}$ denote the expected payoff of player $k$ under separation of control and letting $\prod_k^{IN}$ denote the expected payoff of player $k$ under integration of control $(k = 1, 2, P)$, we use superscripts $SP, IN$, respectively denoting “separation” and “integration”. The payoffs for the respective players are the following.

$$\prod_1^{SP} = \{e_1 b + (1 - e_1)e_2 \gamma b\} + \{e_2 \gamma b + (1 - e_2)e_1 b\} - g(e_1) \quad \ldots(3)$$

$$\prod_2^{SP} = \{e_1 \gamma b + (1 - e_1)e_2 b\} + \{e_2 b + (1 - e_2)e_1 \gamma b\} - g(e_2) \quad \ldots(4)$$

$$\prod_P = \{e_1 \alpha B + (1 - e_1)e_2 \alpha B\} + \{e_2 \alpha B + (1 - e_2)e_1 \alpha B\} \quad \ldots(5)$$

$$\prod_1^{IN} = 2\{e_1[\theta \beta + (1 - \theta) b] + (1 - e_1)e_2 \tau b\} - g(e_1) \quad \ldots(6)$$

$$\prod_2^{IN} = 2\{e_1[\theta \beta + (1 - \theta) \gamma b] + (1 - e_1)e_2 tb\} - g(e_2) \quad \ldots(7)$$

$$\prod_P^{IN} = 2\{e_1[\theta \beta + (1 - \theta) \theta B] + (1 - e_1)e_2 \tau \alpha B\} \quad \ldots(8)$$

The levels of effort in equilibrium are given by the following first-order conditions.

$$g'(e_1^{IN}) = 2b(\theta \beta + 1 - \theta + t e_2^{IN}) \quad \ldots(9)$$

$$g'(e_2^{IN}) = 2tb(1 - e_2^{IN}) \quad \ldots(10)$$

$$g'(e_1^{SP}) = b[2 - (1 + \gamma)e_2^{SP}] \quad \ldots(11)$$

$$g'(e_2^{SP}) = b(2 - (1 + \gamma)e_2^{SP}) \quad \ldots(12)$$

It is apparent that the efforts of agents are strategic substitutes in this case, which implies that if one agent exerts higher effort and is informed, then the other has less incentive to search because the agent might receive the nonnegative payoff for no effort.

4.1 Level of Effort in Equilibrium

We now specify the cost function of effort so that $g(e) = be^\gamma$, which is used by Bao and Wang (2012). When $1 - \theta(1 - \beta) - t \gamma \geq 0$ is satisfied, the value of right-hand side of the equations (9)-(12) is between 0 and 2b. Therefore, we can assume for simplicity that the effort levels in equilibrium are always between 0 and 1. We can ascertain their specified effort levels from simple calculation:
Next we consider the property of the effort levels above. Four related observations about the effort levels must be made. First, we consider the case without intrinsic motivation. If $\theta = 0$, then the agent has no intrinsic motivation under integration. Then, the efforts are $e_1^{IN} = 1, e_2^{IN} = 0$. Worker $A_1$, if informed, can choose a favorite project in the two divisions. Therefore, that worker has much higher incentive to search. Worker $A_2$ saves the cost of effort by free riding on the information of $A_1$ (strategic substitute). This result obtained under the case of $\theta = 0$ is the same as that reported by Bao and Wang (2012).

Second, we investigate the interaction between the effort levels and probability $t$, with which $A_1$ approved $F_2$ proposed by $A_2$. We obtain $\partial e_1^{IN} / \partial t < 0, \partial e_2^{IN} / \partial t > 0$ from simple calculation. One might intuit that $A_2$ exerts higher effort to propose his favorite project ($F_2$) because $A_1$’s screening ability is higher. Then, from strategic substitute, $e_1^{IN}$ is decreasing. We have the following lemma:

**Lemma 1**: The agent in control exerts less effort, while the agent losing control exerts greater effort under a higher level of the agent’s ability ($t$).

Third, we examine the interaction between the effort levels and the extent of intrinsic motivation $\theta$. We have $\partial e_1^{IN} / \partial \theta < 0, \partial e_2^{IN} / \partial \theta > 0$. It is apparent that the agent with high intrinsic motivation exerts less effort because choosing the firm management’s favorite project gives the worker less payoff after all. Then, worker $A_2$ exerts more effort from a strategic substitute. We therefore have the following.

**Lemma 2**: The agent in control exerts less effort, whereas the agent losing control exerts greater effort with a higher level of the extent of intrinsic motivation ($\theta$).

Finally, we check the large/small relation between $e_1^{IN}$ and $e_2^{IN}$. It is apparent that

$$e_1^{IN} \leq e_2^{IN} \iff \overline{\theta} \equiv \frac{1-\gamma t^2}{(1-\beta)(1+t)} \leq \theta.$$  

Consequently, in the case in which the screening ability ($t$) is sufficiently large to for $\overline{\theta}$ to be between 0 and 1, when $\theta$ is between $\overline{\theta}$ and 1, $e_1^{IN} < e_2^{IN}$ holds: The effort exerted by the agent in control is lower than the effort exerted by the agent losing control. The reasons are the following. First, when $\theta$ is sufficiently large, $A_1$ loses incentive to search from lemma 2. Second, when $t$ is sufficiently large,
A_2 exerts greater effort level from lemma 1. The result contrasts with that reported by Bao and Wang (2012), where the agent having two division’s control chooses a maximized effort level and the agent losing control chooses minimized effort level (i.e. e^{IN}_1 = 1, e^{IN}_2 = 0). We have the following proposition:

**Proposition 1**: If the screening ability of A_i and the extent of intrinsic motivation is sufficiently large, then the effort chosen by the agent having control is less than the effort exerted by the agent losing control.

### 4.2 Optimal Task Design

We now derive the firm’s expected payoffs in both integration and separation and consider the optimal task design for the firm. Substituting (13)-(15) into (5) and (8), respectively, the firm’s payoffs are calculated as shown below.

\[
\Pi^{IN}_p = \frac{2(1 - \gamma t^2)[\alpha + (1 - \alpha)\theta][1 - \gamma t^2 - (1 - \beta)\theta] + \alpha(1 - \beta)^2 \alpha^2 t^2}{(1 - \gamma t^2)^2} B \quad \ldots(16)
\]

\[
\Pi^{SP}_p = \frac{8\alpha(2 + \gamma) B}{(3 + \gamma)^2} \quad \ldots(17)
\]

We first examine the optimal task design when the agent in control has no intrinsic motivation and always approves the project proposed by the agent without control (i.e. \( \theta = 0, t = 1 \)). Then, the difference between (16) and (17) is the following.

\[
\Pi^{IN}_p - \Pi^{SP}_p = \frac{(1 + \gamma)^2}{(3 + \gamma)^2} > 0
\]

Therefore, it is always optimal for the firm management to choose integration in this case because the firm management can obtain the payoff with certainty under integration: the agent in control exerts the maximum level of effort.

Next, we now consider a case in which there exists both intrinsic motivation and screening ability. Therefore, \( \theta, t \in (0, 1) \). It is difficult to calculate the difference between the respective payoffs. We therefore use the following parameter specification: \( \alpha = \frac{1}{3}, \beta = \gamma = \frac{1}{2} \) (Case 1) and \( \alpha = \frac{2}{3}, \beta = \gamma = \frac{1}{2} \) (Case 2). We can establish the following proposition:

**Proposition 2**: In Case 1 \( \left( \alpha = \frac{1}{3}, \beta = \gamma = \frac{1}{2} \right) \) we always have \( \Pi^{IN}_p > \Pi^{SP}_p \). In Case 2 \( \left( \alpha = \frac{2}{3}, \beta = \gamma = \frac{1}{2} \right) \) we obtain the following results:

(1) when the screening ability is higher \( \left( \sqrt{\frac{23 + \sqrt{7497}}{134}} \leq t < 1 \right) \) it is apparent that \( \Pi^{IN}_p > \Pi^{SP}_p \), and

\( A_2 \)
(2) when the screening ability is lower \(0 < t < \sqrt[134]{23 + \sqrt{7497}}\), \(\Pi_{P}^{IN} > \Pi_{P}^{SP}\) holds under \(0 < \theta \leq \bar{\theta}\) and \(\Pi_{P}^{IN} > \Pi_{P}^{SP}\) holds under \(\hat{\theta} < \theta < 1 (\exists \hat{\theta} \in (0, 1))\).

In Case 1, where \(\alpha\) is lower than in Case 2, integration is always optimal \((\Pi_{P}^{IN} > \Pi_{P}^{SP})\). If \(A_{1}\) chooses the firm’s favorite project under integration, then the firm can receive \(B\). However, under separation, the firm’s payoff is at most \(\alpha B\) because \(A_{1}\) does not implement the firm management’s favorite project. Consequently, in the case in which \(\alpha\) is lower, it is better for the firm to adopt integration.

In Case 2, we consider the case in which \(\alpha\) is higher than that of Case 1. Then, when \(t\) is sufficiently large, integration is always an optimal task design. However, particularly when \(t\) is sufficiently small and \(\theta\) is sufficiently large, separation might be optimal. The reasons are the following. First, if \(t\) is small, then \(A_{1}\) frequently rejects the project proposed by \(A_{2}\). Consequently, the firm does not receive positive payoffs. Second, if \(\theta\) is large, then \(A_{1}\) has less of an incentive to search according to lemma 2. Consequently, the probability of implementing some project decreases. The result contrasts with that reported by Bao and Wang (2012), which presents optimality of the firm to choose integration in which the agent in control exerts the maximum effort level.

5. Conclusion

As described in this report, we show how the intrinsic motivation of workers affects the decision of whether to choose either integration or separation with an incomplete contract. To consider that, we introduce intrinsic motivation and the screening ability to the model of Bao and Wang (2012).

Results show that when intrinsic motivation is high and the screening ability is low, a firm might choose separation, in contrast to results reported by Bao and Wang (2012), where the principal invariably adopts integration (Proposition 2). An agent with high intrinsic motivation exerts low effort level (Lemma 2). The agent in control often rejects the project proposed by the agent losing control under a low level of screening ability. Then, the efficiency of integration discussed by Bao and Wang (2012) is lost.

The result presented herein has implications for optimal task design in a firm. The firm usually assigns many integrated tasks to the worker who is intrinsically motivated because the worker works for the firm management’s profit. However, once the worker notices that altruistic behavior yields a lower personal return, the worker might lose incentive to undertake greater effort. Then, integration is not always optimal for the firm management. Recently, great discussion has arisen about prolonged work as a social problem in Japan. Long work hours can cause an employee’s death from overwork: Karoshi. One cause
of the prolonged work hours is that too many tasks are integrated into the schedule of one worker. Our result indicates that it is favorable in terms of firm's profit to separate the tasks in the case conducted explained above. This can be regarded as a decrease in the length of working time.

Further study of the interaction between intrinsic motivation and the task design problem is expected to be of value to the field of Human Resource Management. We have assumed that the extent of intrinsic motivation is defined as the probability of choosing the firm's favorite project. If we alter the definition of intrinsic motivation, then we can get additional implications in Human Resource Management. For instance, we were able to investigate the case in which motivated agents intrinsically derive utility from exerting effort, as described by Delfgaauw and Dur (2008). These tasks are left as subjects for future research.

References


