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Oscillatory Effect on Magneto - Hydrodynamic Flow and Heat Transfer in a Rotating Horizontal Porous Channel

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Abstract – The present paper studies the effects of injection/suction on an oscillatory flow of a viscous incompressible fluid in a porous channel under the influence of a magnetic field acting normally to the plates of channel. The Channel with variable temperature and constant injection/suction rotates about an axis perpendicular to the plates of channel. The upper plate of the channel is allowed to oscillate in its own plane whereas the lower plate is held at rest. The heat transfer is analyzed under two conditions, the temperature of upper plate of channel starts oscillations whereas the temperature of lower plate is kept fixed. It is interesting to note the magnetic fields fixed relative to the moving plate contribute more to the resultant velocity than the magnetic field fixed relative to the fluid. Another striking result is that the frequency of oscillation has a distinct effect when the magnetic field is fixed relative to fluid. The effect of all pertinent parameters on phase angle is just opposites to that of resultant velocity owing to the relative position of magnetic fields.

Keywords – Heat transfer, Injection/Suction, Porous Medium, Rotation Oscillatory Flow, MHD Flow.

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1. INTRODUCTION

The rotating flow of electrically conducting viscous incompressible fluid in hydrodynamics has achieved considerable attention owing to its several applications in engineering and physics. In geophysics it is utilized to measure and study the positions and velocities with respect to a fixed co-ordinate system on the surface of earth rotating with regards to an inertial frame in the presence of its magnetic field. The general notion of fluid flow and mass transfer past a porous medium in a rotating environment plays a very significant role in the applications of petrochemical engineering, oceanography, aeronautics, meteorology and geophysics. The motivation for scientific investigation on rotating fluid system is fundamentally originated from geophysical and fluid engineering applications. The rotational fluid theory is applied to determine the viscosity of fluid, in the preparation of turbine and other numerous centrifugal machines.

The study of the theory of rotating fluids due to its occurrence in different natural phenomena and its applications in several technological situations is an urgent need which is directly governed by coriolis force. The wide- spread subject matters of atmospheric science; limnology and oceanography contain some features of great importance in rotating fluids. Numerous researchers like Siegman[1], Ganapathy [2] , Guria [3], Hayat and Hutter[4] and Mazumder[5] have studied on hydrodynamic flow of viscous incompressible electrically conducting fluid in a rotating medium. The investigation of magneto-hydrodynamic fluid flow of a viscous incompressible electrically conducting rotating fluid has been studied by several authors viz. Hayat and Abelman [6], Ghosh and Pop [7] under different conditions and configurations to describe various aspects of the problem concerned. Reddy et.al [8] have analyzed heat transfer in hydro magnetic rotating viscous fluid flow through non- homogeneous porous medium with constant heat source/ sink. Raju et. al. [9] have analyzed an unsteady MHD radiative chemically reactive and rotating fluid past an impulsively started vertical plate with variable temperature and mass diffusion. Philip et.al. [10] Studied MHD rotating heat and mass transfer free convective flow past an exponentially accelerated isothermal plate with fluctuating mass diffusion. Hari krishna et.al. [11] have thrown light upon Hall current effects on an unsteady MHD flow in a rotating parallel plates channel bounded by porous bed on the lower half Darcy Lapwood model. Reddy et. al. [12] studied thermal diffusion and rotating effects on MHD mixed convective flow of heat absorbing/generating visco-elastic fluid through a porous channel.

Singh [13] and Seth et.al [14] explored oscillatory MHD couette flow of incompressible electrically conducting viscous fluid in a rotating system under various conditions. Guria et.al [15] considered oscillatory MHD couette flow of electrically conducting fluid between two parallel plates in a rotating system under influence of inclined magnetic field in which the upper plate is kept fixed at rest and the lower plate oscillates non torsionally. Das et.al. [16] have investigated unsteady hydro magnetic couette flow of incompressible viscous and electrically conducting fluid in the rotating system when the flow of fluid within the channel is deduced on account of impulsive movement of one of the plates of the channel. Singh et.al.[17] have studied unsteady hydro magnetic couette flow of viscous incompressible electrically conducting fluid in the rotating system between two parallel plates when one of the plates of the channel is kept uniformly accelerated. Further, Seth et.al. [18] discussed MHD couette flow problem when the lower plate of the channel moves with time dependent velocity and upper plate is held fixed. They investigated two special cases of huge interest. One for impulsive movement of plate and second for uniformly accelerated movement of the plate. In all these investigations, the walls of channels are supposed to be non- porous.

The study of fluid flow through porous channels have numerous engineering and geophysical applications in the areas of chemical engineering for the sake of purification and filtration process , in petroleum technology to estimate the movement of natural gas through channel, In agriculture engineering to study the underground water resources, minerals and metallurgical industries, designing of cooling systems together with liquid metals, geothermal reservoirs, underground energy transport , MHD pumps and generators, Accelerators and flow meters etc. Keeping in view of these applications Singh and Sharma [19] studied influence of permeability of porous medium on three dimensional couette flow and heat transfer. Hayat et.al. [20], Singh [21] investigated MHD flow within a parallel channel with porous boundaries under different conditions in non-rotating system. Ahmadi and Manvi [22] studied unsteady MHD flow of conducting fluid through porous medium. Ram and Mishra[23] applied the equations of motion derived by above Ahmadi and Manvi for the purpose of investigation on the MHD flow through porous medium and obtained different results. M.M. Hamza et. al . [24] Studied unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition. K.D. Singh [25] studied the effect of slip condition on visco-elastic MHD oscillatory forced convection flow in a vertical channel with heat radiation. J.L. Flade et.al. [26] Investigated in influence of suction or injection on MHD oscillatory flow through the porous channel saturated with porous medium. K. M. Joseph [27] studied the chemical reacting fluid on unsteady MHD oscillatory slip flow in planer channel with varying temperature and concentration in the presence of suction or injections.

The purpose of the present analysis is to study the effect of oscillation on the flow and heat transfer phenomena of a viscous incompressible electrically conducting fluid in a rotating horizontal porous channel in the presence of uniform transverse magnetic field when the field is fixed relative to the fluid or to the moving plate. The plates of channel are supposed to be porous and the flow within the channel is owing to the oscillatory motion of upper plate. This model has significant utilizations in novel MHD energy system, magneto biofluid and designing MHD devices requiring control of fluid flow.

2. ANALYSIS OF MATHEMATICAL MODEL

Here, we consider an oscillatory flow of electrically conducting viscous- incompressible fluid confined between two separated parallel porous plates of infinite length at a distance ‘h’ apart under the influence of a uniform transverse magnetic field B0 applied in the direction parallel to Z' - axis perpendicular the planes of the plates. We consider here two situations of magnetic fields, firstly magnetic field is fixed with respect to fluid and secondly the magnetic field is fixed relative to the moving plate. The heat transfer case of fluid is also studied. A constant injection velocity v0 is applied at the upper oscillating plate as well as lower stationary plate. The upper plate oscillates in its own plane with a velocity $U_0(1+\in cosw't')$ about a non-zero uniform mean velocity U0 .

The plates of the channel as well as fluid are in the situation of a rotation of a rigid body with constant angular velocity Ω about z'- axis. Take the origin at the lower plate lying in x'- y' plane. The axis of x' is chosen in parallel direction of the motion of upper plate. The plates of the channel are infinite in the directions of x' and y'. The plates are electrically non- conductors therefore all physical quantities, except pressure, will be functions of z' and t'. For metallic liquids, the magnetic Reynolds number is small enough so the induced magnetic field may be ignored in comparison to applied magnetic field.

Initially t'< 0, the fluid as well as plates are considered at rest. At the time when t'>0, the upper plate begins moving with a velocity proportional to U' (t') in the reference frame of co-ordinates rotating with fluid. The equations of motion, energy and continuity are given in vector form as

$$\frac{\partial \vec{V}}{\partial t'} + (\vec{V}.\nabla)\vec{V} + 2\Omega\vec{n} \times \vec{V} = -\frac{1}{\rho}\nabla p + \upsilon\nabla\vec{V} + \frac{1}{\rho}\vec{J} \times \vec{B} - \frac{\mu}{\rho K'}\vec{V} \qquad (6.2.1)$$

$$\frac{DT'}{Dt'} = \frac{1}{\rho C_p}\nabla.(k\nabla T') \qquad (6.2.2)$$

$$\nabla.\vec{V}=0 \qquad (6.2.3)$$

With the following results

$$\text{(Ampere's law)} \quad \nabla\times\vec{B}=\mu_e\vec{J} \qquad (6.2.4)$$

$$\text{(Maxwell' law)} \quad \nabla.\vec{B}=0 \qquad (6.2.5)$$

$$\text{(Faraday's law)} \quad \nabla\times\vec{E}=\frac{\partial\vec{B}}{\partial t'} \qquad (6.2.6)$$

$$\text{(Gauss's law)} \quad \nabla.\vec{J}=0 \qquad (6.2.7)$$

$$\text{(Ohm's law)} \quad \vec{J}=\sigma(\vec{E}+\vec{V}\times\vec{B}) \qquad (6.2.8)$$

for a moving conductor without Hall current.

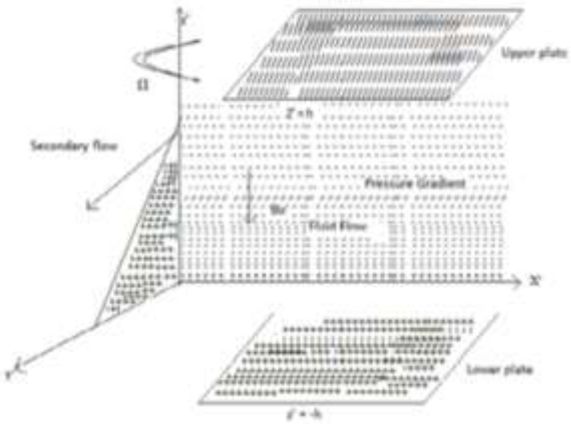

Fig 1 : Reference frame and physical model.

The symbols $\vec{J},\vec{E},\vec{B},\vec{V}$ are defined as current density, electric field , the magnetic field and the velocity of fluid respectively. $\vec{n}$ is the unit vector along z' direction, μ_e is the magnetic permeability, σ is the fluid electric conductivity, t' represents time.

The mathematical model of the problem is depicted in figure 1 , where $\vec{V}=(u'_{x'},v'_{y'},w'_{z'})$ is the velocity vector is x',y', z' directions respectively.

$$\vec{B}=(0,0,B_0),\vec{E}=(E_{x'},E_{y'},E_{z'}),\vec{J}=(J_{x'},J_{y'},J_{z'})$$

such that B0 is a constant. We make here an assumption that no polarization voltage comes into existence i.e. $\vec{E}=0$. Equation (6.2.7) is Gaussian law of conservation for electric charge i.e $\nabla.\vec{J}=0$ which yields Jy' = constant. If we consider the case of vertical component of velocity we ultimately obtain $J_{z'}$ = 0.

Hence equation (6.2.8) immediately gives

$$J_{x'}=\sigma B_0^2 v'_{y'} \text{ and } J_{y'}=-\sigma B_0^2 u'_{x'} \qquad (6.2.9)$$

Taking the above results in view of equations (6.2.1) and (6.2.2) are rewritten in component form as

$$\frac{\partial u'_{x'}}{\partial t'}+w'_{z'}\frac{\partial u'_{x'}}{\partial z'}-2\Omega v'_{y'}=-\frac{1}{\rho}\frac{\partial p'}{\partial x'}+\upsilon\frac{\partial^2 u'_{x'}}{\partial^2 z'^2}-\sigma\frac{B_0^2}{\rho}u'_{x'}-\frac{\mu}{\rho K'}u'_{x'} \ldots \qquad (6.2.10)$$

$$\frac{\partial v'_{y'}}{\partial t'}+w'_{z'}\frac{\partial v'_{y'}}{\partial z'}+2\Omega u'_{x'}=-\frac{1}{\rho}\frac{\partial p'}{\partial y'}+\upsilon\frac{\partial^2 v'_{y'}}{\partial z'^2}-\sigma\frac{B_0^2 v'_{y'}}{\rho}-\frac{\mu}{\rho K'}v'_{y'} \qquad (6.2.11)$$

and

$$\frac{\partial T'}{\partial t'}+w'_{z'}\frac{\partial T'}{\partial z'}=\frac{K'}{\rho C_p}\frac{\partial^2 T'}{\partial z'^2} \qquad (6.2.12)$$

The present problem takes the following initial and boundary condition.

$u'_{x'}=v'_{y'}=0$ for $0\le z'\le h$ and $t'\le 0$

$u'_{x'}=v'_{y'}=0, w'_{z'}=v_0, T'=T_0$ at $z'=0$

$u'_{x'}=U'(t')=U_0(1+\varepsilon\cos\omega't'),\ v'_{y'}=0,\ w'_{z'}=v_0$ at $z'=h$

$T'=T_h$ at $z'=h$

(The constant plate temperature)

$$T'=T_h+\varepsilon(T_0-T_h)\cos\omega't' \text{ at } z'=h \text{ for } t'>0. \qquad (6.2.13)$$

(oscillatory plate temperature)

Here we notice that equation (6.2.10) holds good only when the magnetic field is fixed with regards to fluid. If the magnetic field is kept fixed relative to the moving plate, equation (6.2.10) is replaced following Raptis and Singh [28] by writing as

$$\frac{\partial u'_{x'}}{\partial t'}+w'_{z'}\frac{\partial u'_{x'}}{\partial z'}-2\Omega v'_{y'}=-\frac{1}{\rho}\frac{\partial p'}{\partial x'}+\upsilon\frac{\partial^2 u'_{x'}}{\partial z'^2}-\sigma\frac{B_0^2}{\rho}(u'_{x'}-U')-\frac{\mu}{\rho K'}u'_{x'} \qquad (6.2.14)$$

Now equations (6.2.10) and (6.2.14) can be combined together to get a single equation which as follows:

$$\frac{\partial u'_{x'}}{\partial t'}+w'_{z'}\frac{\partial u'_{x'}}{\partial z'}=-\frac{1}{\rho}\frac{\partial p'}{\partial x'}+2\Omega v'_{y'}-\sigma\frac{B_0^2}{\rho}(u'_{x'}-\lambda U')-\frac{\upsilon}{K'}u'_{x'} \qquad (6.2.15)$$

Where λ is defined as

λ =0 provided B_0 is fixed with respect to fluid. Again λ =1 if B_0 is fixed with regards to moving plate. Here the terms used have the following meaning. K' is the permeability of medium, υ the kinematic viscosity, ρ is the density of fluid, K the thermal conductivity, c_p is the specific heat at constant pressure.

From equation (6.2.3) on integration, it is clear that $w'_{z'}=v_0$ (constant).

We find the relation between modified pressure gradients $\frac{\partial p'}{\partial x'}$ and $\frac{\partial p'}{\partial y'}$ under usual boundary layer approximations on equations (6.2.10) and (6.2.11). Thus, we obtain the relation in the form

$$\frac{\partial U'}{\partial t'} + \sigma \frac{B_0^2}{\rho} U' + \frac{\upsilon}{k'} U' = -\frac{1}{\rho}\frac{\partial p'}{\partial x^2} - 2\Omega' U' = -\frac{1}{\rho}\frac{\partial p'}{\partial y'} \qquad (6.2.16)$$

Substituting $w'_{z'} = v_0$ and values of $\frac{\partial p'}{\partial x'}$ and $\frac{\partial p'}{\partial y'}$ from (6.2.16) into (6.2.11) and (6.2.15) we get

$$\frac{\partial v'_y}{\partial t'} + v_0 \frac{\partial v'_y}{\partial z'} = \upsilon \frac{\partial^2 v'_y}{\partial z'^2} - 2\Omega' (u'_{x'} - U') - \sigma \frac{B_0^2}{\rho} v' - \frac{\upsilon}{k'} v'_y \qquad (6.2.17)$$

$$\frac{\partial u'_x}{\partial t'} + v_0 \frac{\partial u'_x}{\partial z'} = \frac{\partial U'}{\partial t'} + \upsilon \frac{\partial^2 u'_x}{z'^2} + 2\Omega' v'_y + \sigma \frac{B_0^2}{\rho} [u'_{x}{}' - (1+\lambda)U'] - \frac{\upsilon}{k'}(u'_{x'} - U') \quad (6.2.18)$$

To non- dimensionalise the equation (6.2.12), (6.2.17) and (6.2.18) we introduce the following non-dimensional quantities

$\omega = \frac{\omega' h^2}{\upsilon}$ (Frequency parameter)

$M = B_0 h \sqrt{\frac{\sigma}{\mu}}$ (Hartmann number)

$\lambda_0 = \frac{V_0 h}{\upsilon}$ (Injection/Suction parameter)

$\Omega = \frac{\Omega' h^2}{\upsilon}$ (Rotational parameter)

$S_p = \frac{k'}{h^2}$ (Permeability parameter)

$P_r = \frac{k' \upsilon}{\rho c_p}$ (Prandtl number parameter)

$$\zeta = \frac{z'}{h}, t = \omega' t', u = \frac{u'_{x'}}{U_0}, v = \frac{v'_y}{V_0}, T = \frac{T}{T_0}$$

And finally we get

$$\frac{\partial T}{\partial t} + \frac{\lambda_0}{\omega}\frac{\partial T}{\partial \zeta} = \frac{1}{P_r}\omega \frac{\partial^2 T}{\partial \zeta^2} \qquad (6.2.19)$$

$$\omega \frac{\partial v}{\partial t} + \lambda_0 \frac{\partial v}{\partial \zeta} = \frac{\partial^2 v}{\partial \zeta^2} - 2\Omega(u - U) - \left(M^2 + \frac{1}{s_p}\right) v \qquad (6.2.20)$$

And

$$\omega \frac{\partial u}{\partial t} + \lambda_0 \frac{\partial u}{\partial \zeta} = \frac{\partial^2 u}{\partial \zeta^2} + \omega \frac{\partial U}{\partial t} + 2\Omega v - M^2[u - (1+\lambda)U] - \frac{1}{s_p}(u - U) \quad (6.2.21)$$

The transformed corresponding boundary conditions are given by

$$\left.\begin{array}{l} u = v = 0 \text{ at } 0 \le \zeta \le 1, t \le 0 \\ u = v = 0, T = 1, \text{ at } \zeta = 0, \text{ for } t > 0 \\ u = U(t) = 1 + \epsilon \cos t, v = 0 \text{ at } \zeta = 1 \end{array}\right\}$$

For constant plate temperature

$$T = 0 \text{ at } \zeta = 1 \qquad (6.2.22)$$

For oscillating plate temperature

$T = \epsilon \cos t$ at $\zeta = 1$

3. SOLUTION OF THE PROBLEM

By introducing a complex function F = u + iv, equations (6.2.21) and (6.2.20) are combined into a single equation in the form

$$\omega \frac{\partial F}{\partial t} + \lambda_0 \frac{\partial F}{\partial \zeta} = \frac{\partial^2 F}{\partial \zeta^2} + \omega \frac{\partial U}{\partial t} - \left[2\Omega i + M^2 + \frac{1}{s_p}\right] + \left[2\Omega i + M^2(1+\lambda) + \frac{1}{s_p}\right] \qquad (6.3.1)$$

The boundary condition (6.2.22) for F can be given by

$$F = 0 \text{ for } 0 \le \zeta \le 1, t \ge 0$$

$$F = 0 \text{ for } \zeta = 0, t > 0 \qquad (6.3.2)$$

$$F = U(t) = 1 + \frac{\epsilon}{2}\left(e^{it} + e^{-it}\right) \text{ at } \zeta = 1, t > 0$$

We consider the solutions following Light hill [29] in the following form

$$F(\zeta, t) = F_0(\zeta) + \frac{\epsilon}{2}[F_1(\zeta)e^{it} + F_2(\zeta)e^{-it}] \qquad (6.3.3)$$

And

$$T = T_0(\zeta) + T_1(\zeta)\, e \cos t \qquad (6.3.4)$$

Putting (6.3.3) and (6.3.4) in (6.3.1) and (6.2.19) respectively and comparing harmonic and non-harmonic terms we obtain

$$\frac{d^2 F_0}{d\zeta^2} - \lambda_0 \frac{dF_0}{d\zeta} - \left(2i\Omega + M^2 + \frac{1}{s_p}\right) F_0 = -\left[2i\Omega + (1+\lambda)M^2 + \frac{1}{s_p}\right] \qquad (6.3.5)$$

$$\frac{d^2 F_1}{d\zeta^2} - \lambda_0 \frac{dF_1}{d\zeta} - [i(2\Omega + \omega) + M^2 + \frac{1}{s_p}]F_1 = -\left[i(2\Omega + \omega) + (1+\lambda)M^2 + \frac{1}{s_p}\right] \qquad (6.3.6)$$

$$\frac{d^2 F_2}{d\zeta^2} - \lambda_0 \frac{dF_2}{d\zeta} - [i(2\Omega - \omega) + M^2 + \frac{1}{s_p}]F_2 = -\left[i(2\Omega - \omega) + (1+\lambda)M^2 + \frac{1}{s_p}\right] \qquad (6.3.7)$$

$$\frac{d^2 T_0}{d\zeta^2} - P_r \lambda_0 \frac{dT_0}{d\zeta} = 0 \qquad (6.3.8)$$

$$\frac{d^2 T_1}{d\zeta^2} - P_r \lambda_0 \frac{dT_1}{d\zeta} + (P_r \omega \tan t) T_1 = 0 \qquad (6.3.9)$$

The relevant corresponding boundary conditions and two different cases of temperature become

$$\left.\begin{array}{l} F_0 = F_1 = F_2 = 0 \text{ for } 0 \le \zeta \le 1,\ t \ge 0. \\ F_0 = F_1 = F_2 = 0\ T_0 = 1,\ T_1 = 1 \text{ for } \zeta = 0. \end{array}\right\}$$

$$F_0 = F_1 = F_2 = 1 \text{ at } \zeta = 1$$

$T_0 = 0$, $T_1 = 0$ for $\zeta = 1$ constant plate temperature

$T_0 = 0$, $T_1 = 1$ for $\zeta = 1$ oscillatory plate temperature for t>0 (6.3.10)

The solution of equations (6.3.5) - (6.3.9) are obtained under the boundary conditions (6.3.10) as

$$F_0 = \frac{\exp(s_2+s_1\zeta) - \exp(s_1+s_2\zeta)}{\exp(s_1) - \exp(s_2)} + A_1 \qquad (6.3.11)$$

$$F_1 = \frac{\exp(s_4+s_3\zeta) - \exp(s_3+s_4\zeta)}{\exp(s_3) - \exp(s_4)} + A_2 \qquad (6.3.12)$$

$$F_2 = \frac{\exp(s_6+s_5\zeta) - \exp(s_5+s_6\zeta)}{\exp(s_5) - \exp(s_6)} + A_3 \qquad (6.3.13)$$

Where

$$s_1 = \frac{\lambda_0 + \sqrt{\lambda_0^2 + 4(2i\Omega + M^2 + \frac{1}{sp})}}{2}$$

$$s_2 = \frac{\lambda_0 - \sqrt{\lambda_0^2 + 4(2i\Omega + M^2 + \frac{1}{sp})}}{2}$$

$$s_3 = \frac{\lambda_0 + \sqrt{\lambda_0^2 + 4[i(2\Omega + \omega) + M^2 + \frac{1}{sp}]}}{2}$$

$$s_4 = \frac{\lambda_0 - \sqrt{\lambda_0^2 - 4[i(2\Omega + \omega) + M^2 + \frac{1}{sp}]}}{2}$$

$$s_5 = \frac{\lambda_0 + \sqrt{\lambda_0^2 + 4[i(2\Omega - \omega) + M^2 + \frac{1}{sp}]}}{2}$$

$$s_6 = \frac{\lambda_0 - \sqrt{\lambda_0^2 + 4[i(2\Omega - \omega) + M^2 + \frac{1}{sp}]}}{2}$$

$$A_1 = 1 + \frac{\lambda M^2}{2i\Omega + M^2 + \frac{1}{sp}}$$

$$A_2 = 1 + \frac{\lambda M^2}{i(2\Omega + \omega) + M^2 + \frac{1}{sp}}$$

$$A_3 = 1 + \frac{\lambda M^2}{i(2\Omega - \omega) + M^2 + \frac{1}{sp}}$$

and

$$T_0(\zeta) = \frac{\exp(\lambda_0 P_r) - \exp(\lambda_0 P_r \zeta)}{\exp(\lambda_0 P_r) - 1} \qquad (6.3.14)$$

$$T_1(\zeta) = \frac{exp\frac{(\lambda_0 P_r + \beta)}{2}\zeta - exp\frac{(\lambda_0 P_r - \beta)}{2}\zeta}{exp\frac{(\lambda_0 P_r + \beta)}{2} - exp\frac{(\lambda_0 P_r - \beta)}{2}} \qquad (6.3.15)$$

$$T(\zeta) = \frac{\exp(\lambda_0 P_r) - \exp(\lambda_0 P_r \zeta)}{\exp(\lambda_0 P_r) - 1} + \varepsilon \cos t\ \frac{exp\frac{(\lambda_0 P_r + \beta)}{2}\zeta - exp\frac{(\lambda_0 P_r - \beta)}{2}\zeta}{exp\frac{(\lambda_0 P_r + \beta)}{2} - exp\frac{(\lambda_0 P_r - \beta)}{2}}$$

Where

$$\beta = \sqrt{\lambda_0^2 P_r^2 + 4 P_r \omega\, tant} \qquad (6.3.16)$$

Now our purpose is to consider the resultant velocities and shear stresses of the steady and unsteady motions of flows of fluid. For the sake of analysis, we write

$$F_0(\zeta) = u_0(\zeta) + i\, v_0(\zeta) \qquad (6.3.17)$$

And

$$F_1(\zeta)\exp(it) + F_2(\zeta)\exp(it) = u_1(\zeta) + i\, v_1(\zeta) \qquad (6.3.18)$$

where (u_0, u_1) and (v_0, v_1) are primary and secondary components of velocities of steady and unsteady flow respectively. The expressions for resultant velocities and phase differences for steady and unsteady flows are given by

$$R_0 = \sqrt{u_0^2 + v_0^2} \quad \text{and} \quad \theta_0 = \tan^{-1}\left(\frac{v_0}{u_0}\right) \qquad (6.3.19)$$

And

$$R_1 = \sqrt{u_1^2 + v_1^2} \quad \text{and} \quad \theta_1 = \tan^{-1}\left(\frac{v_1}{u_1}\right) \qquad (6.3.20)$$

respectively.

Further, the shear stress at the stationary plate for the steady flow is given as

$$\tau_{0x} + i\,\tau_{0y} = \left(\frac{\partial F_0}{\partial \zeta}\right)_{\zeta=0}$$

The amplitude and phase difference are written as

$$\tau_{0r} = \sqrt{\tau_{0x}^2 + \tau_{0y}^2},\ \theta_{0r} = \tan^{-1}\left(\frac{\tau_{0y}}{\tau_{0x}}\right) \qquad (6.3.21)$$

Again for the case of unsteady flow, the shear stress at the stationary plate is governed by

$$\left(\frac{\partial u_1}{\partial \zeta}\right)_{\zeta=0} + i\left(\frac{\partial v_1}{\partial \zeta}\right)_{\zeta=0} = \left(\frac{\partial F_1}{\partial \zeta}\right)_{\zeta=0}\exp(it) + \left(\frac{\partial F_2}{\partial \zeta}\right)_{\zeta=0}\exp(-it) = \tau_{1x} + i\tau_{1y} \qquad (6.3.22)$$

The amplitude and phase difference are written as

$$\tau_{1r} = \sqrt{\tau_{1x}^2 + \tau_{1y}^2}\,,\ \theta_{1r} = \tan^{-1}\left(\frac{\tau_{1y}}{\tau_{1x}}\right) \qquad (6.3.23)$$

4. RESULTS AND DISCUSSION

In this section we have discussed the effects of several parameters involved in the present problem graphically under two different cases (a) steady flow case and (b) unsteady flow case.

(a) Steady Case

Figure (2) reveals the influence of rotational parameter Ω on the resultant velocity R_o curve 6 exhibits the effect without rotation (Ω=0) and without porous medium (k_s =0). In this situation, the resultant velocity attains the lowest velocity value in both the cases λ =0 and λ =1. Again, we compare curve-6 (k_s =100), with curve -1(k_s =1), it is clear that the presence of porous matrix without rotation increases R_o. Further, we find that the resultant velocity R_o increases with the increase in the value of rotational parameter Ω. Again observing carefully, it is found that an increase in rotation of frame of reference, the difference between resultant velocities with respect to λ=0 and λ =1 diminishes and ultimately it coincides with the curve-4.

Therefore it may be decided that with higher value of Ω the relative status of magnetic field with regards to moving plate and the fluid has no influence.

The fig 3 exhibits the influence of permeability parameter (s_p) and Hartmann number (M) on the resultant velocity R_o. It is noticed that the resultant velocity R_o remains unaffected by s_p but an increase in M increases R_o. Here it is remarkable to notice that the magnetic field fixed with respect to the moving plate (λ =1) gives more to the resultant velocity than the magnetic field fixed relative to fluid (λ =0).

The figure 4 depicts the effects of suction/injection parameter (λ_0) onresultant velocity R_o by curve-2. It is observed that a significant increase in R_0 is received on account of suctions (λ <5) as shown in curve – 3 and decreases (λ_0 >5) in curve -2.Therefore we infer that the increasing values of λ_0 or decreasing values of λ_0 leads to decrease and increase the resultant value of R_o. It is worth mentioning that resultant velocity R_o for magnetic field fixed relative to the moving plate (λ =1) is always greater than its counter part that is the magnetic field fixed relative to the fluid (λ =0). Again another significant outcome is that in presence or absence of suction/injection, porous media has no valuable effect on the resultant velocity R_o for both λ = 0 and λ =1.

Fig 5, exhibits the change of phase angle θ_0 for different values of λ_0 and Ω. One most striking result is that the phase angle gets negative value for higher value of rotation parameter (Ω ≥ 25) for the layers (ζ>0.4), a little away from the lower plate. Further, the negative value of phase angle is obtained in case of suction (λ_0 =4) and without suction that is for impermeable wall. It is observed that the role of s_p (permeability parameter) is to decrease the phase angle in both the situation λ =0 and λ = 1.

The figure 6 depicts the variation of phase angle θ_0 for different values of parameters s_p and M. It is noticed that the phase angle steadily decreases for different values permeability parameter λ_0. Relative position of magnetic field with respect to fluid (λ =0) has greater phase angle in all cases than its counterpart (λ =0) which has opposite influence on resultant velocity. It is also noted that as the magnetic field enhances, the phase angle increases significantly.

(b) Unsteady Case

Plots 6(a),6(b),6(c) illustrate the resultant velocity R_1 in the case of unsteady fluid motion. On comparing with steady case of motion, it is observed that the influence of parameters s_p, λ_0 ,Ω & M involved in the problem remain invariably the same as that of steady flow motion case.

In figure 8 the influence of frequency of oscillation ω is observed resultant velocity R_1. It is seen that frequency parameter ω has remarkable effect on magnetic field which is fixed relative to the fluid (λ =0). But in case of its counterpart (λ =1) there is no noticeable effect. Moreover, it is observed that the resultant velocity R_1 decreases with respect to increasing value of frequency ω for the cases λ =0 and λ =1.

Fig. 9 and fig. 10 explain the variation of phase angle θ_1 for unsteady flow motion for different values of different parameters like s_p,Ω, λ_0 and M involved in the problem. It is seen that the effect of these parameters on the phase angled remains unchanged in case of steady as well as unsteady motions except the magnitude which is approximately twice than the steady case.

The figure - 11 depicts the changing of phase angle θ_1 for various values of frequency parameter ω. It is observed that the phase angle θ_1 assumes higher values of magnetic field which is fixed relative to the fluid for all values of parameters s_p, λ_0, ω ,Ω and M. It is also significant to note that permeability of medium is responsible for decreasing phase angle whether oscillation is present (ω≠0) or absent (ω=0).

Fig.12 concerns with temperature distribution for different values of parameters like Prandtl number P_r, frequency of oscillationω, and suction parameter λ_0 when the plates are kept at constant temperature. It is observed that higher Prandtl number P_r in the presence of injection give contribution to increase the thickness of thermal boundary layer. On the other hand the suction decreases the thermal boundary layer. Further, it is noticed that frequency ω of oscillation has no influence over thermal boundary layer.

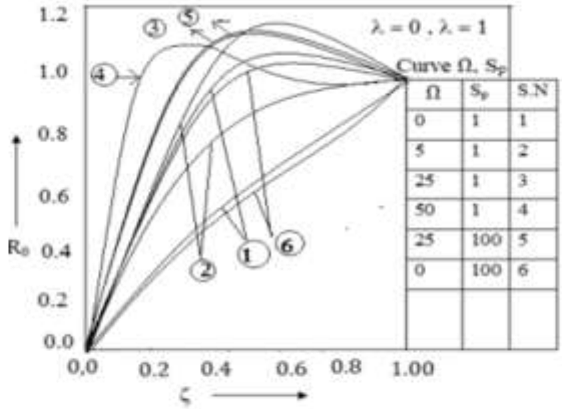

Fig 2 Resultant velocity R_0 for various values of s_p and Ω, M = 2, $\lambda_0 = 2$.

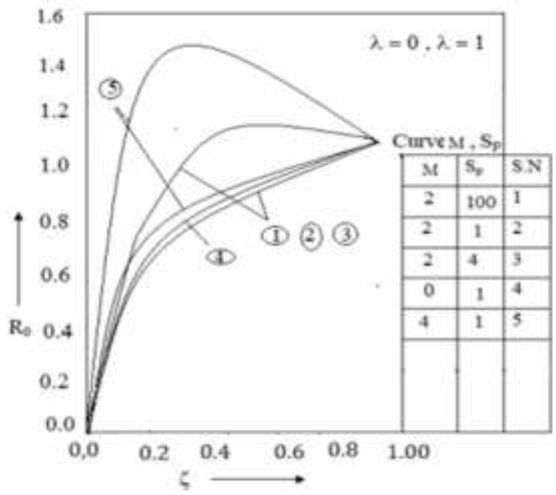

Fig 3 Resultant velocity R_0 for various values of M and S_p . $\lambda_0 = 2$ $\Omega = 5$.

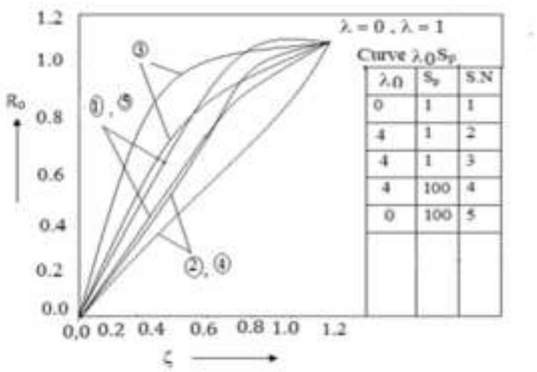

Fig 4 Resultant velocity R_0 for various values of S_p and λ_0 , $\Omega = 5$, M = 2.

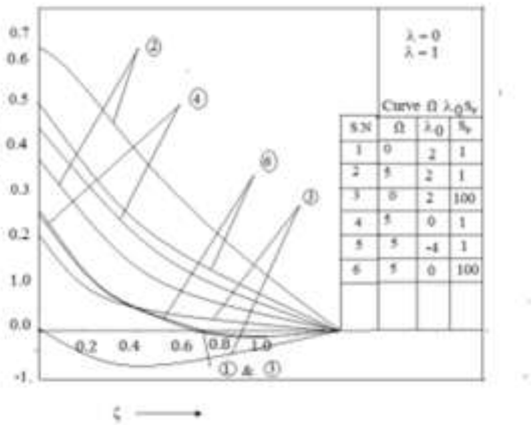

Fig 5 Phase angle θ_0 for various values of S_p , λ_0 and Ω. M = 2

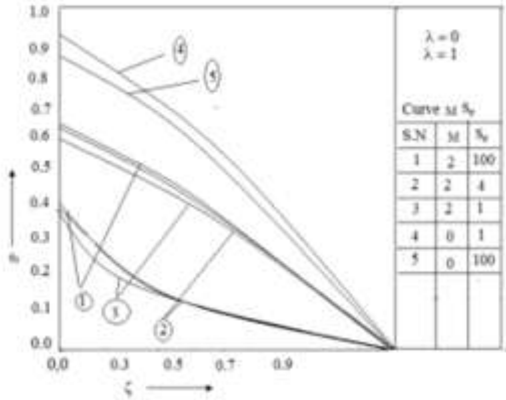

Fig 6 Phase angle θ_0 for various values of S_p and M. $\Omega = 5$, $\lambda_0 = 2$.

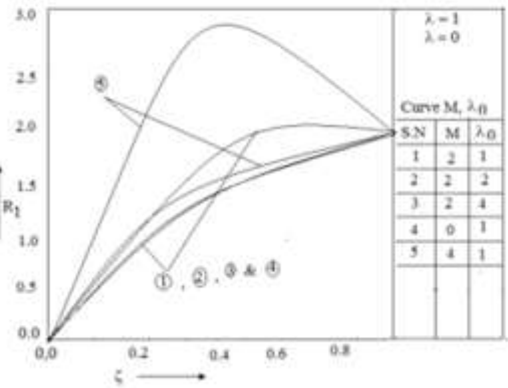

Fig 6 (a) Resultant velocity R_1 for various values of M, S_p, $\Omega = 5$, $\lambda_0 = 2$, $\omega = 5$.

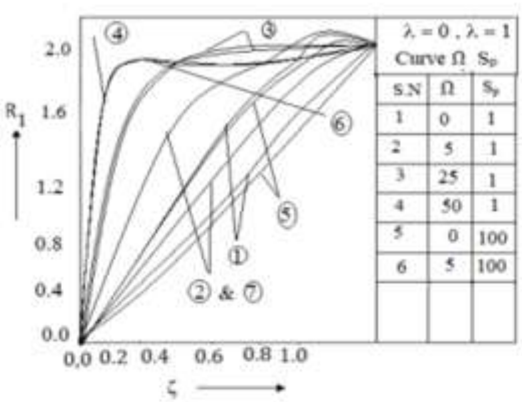

Fig 6 (b) Resultant velocity R_1 for various values of S_p and Ω , M = 2, λ_0 = 2, ω = 5.

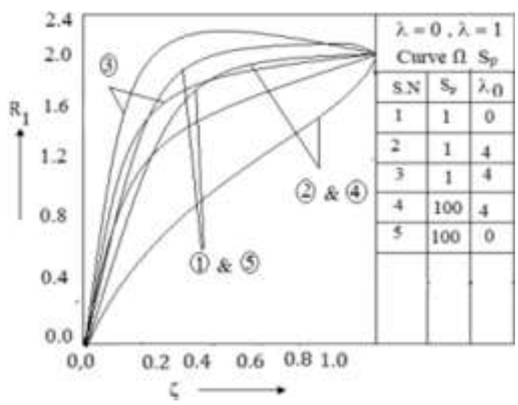

Fig 6 (c) Resultant velocity R_1 for various values of S_p and λ_0 ,Ω =5, M = 2, = 2, ω = 5.

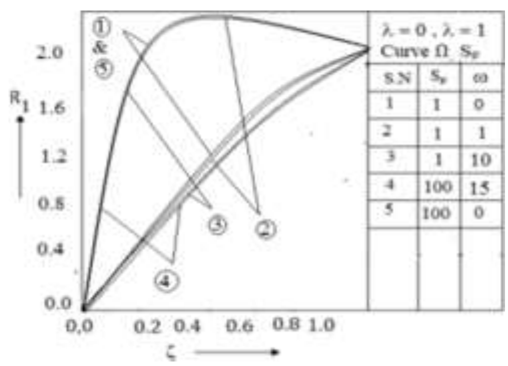

Fig 7 Resultant velocity R_1 for various values of S_p ,ω. Ω =5,λ_0 =2, M = 2.

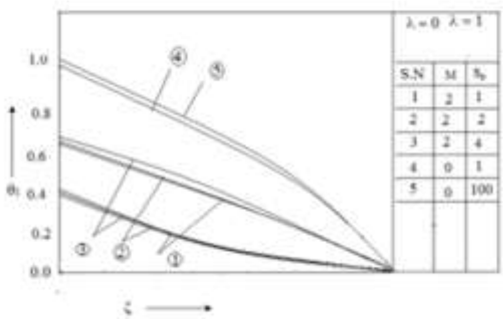

Fig 8 Phase angle θ_1 for various values of M and s_p Ω =5, λ_0 =2, M = 2.

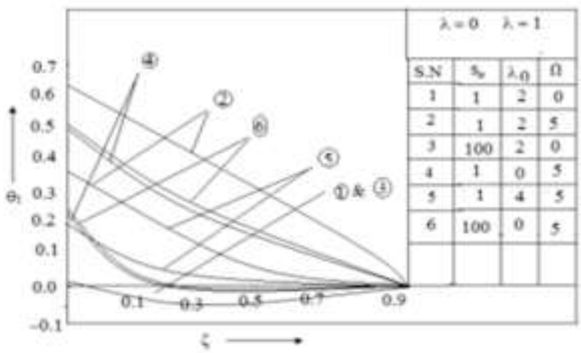

Fig 9 Phase angle θ_1 for various values of parameters s_p , λ_0 and Ω. M = 2,ω=5.

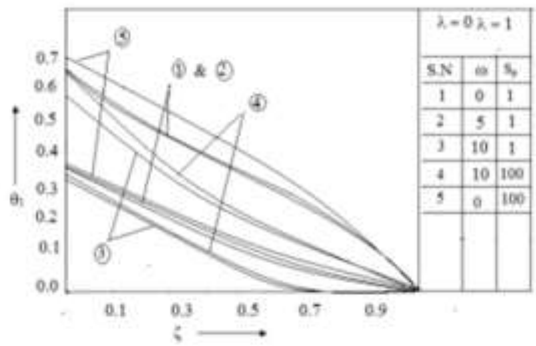

Fig 10 Phase angle θ_1 for various values of parameters s_p , ω,Ω=5 λ_0 =2, M = 2.

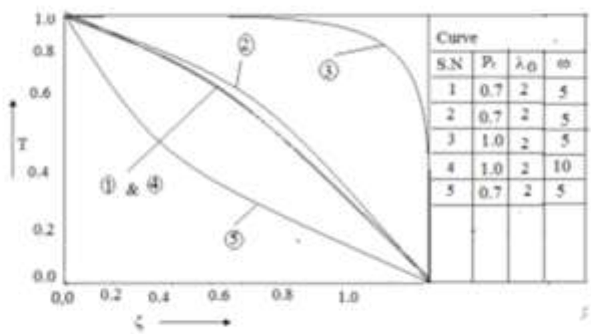

Fig 11Constant temperature distribution for various values of P_r , λ_0 and ω.

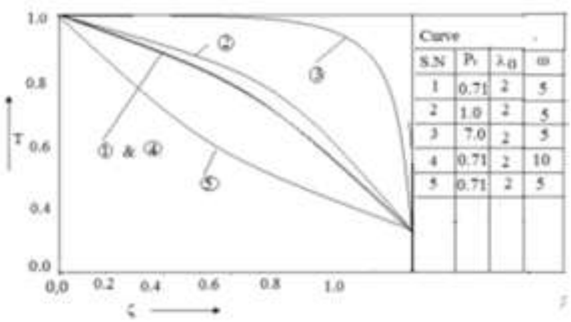

Fig 12 Oscillatory plate temperature distribution for various values of P_r , λ_0 and ω.

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