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Applications of Atangana–baleanu fractional operators in modeling Memory-dependent physical and engineering systems

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Abstract: Atangana-Baleanu fractional operators, with their non-singular and non-local Mittag-Leffler kernel, represent a powerful tool for modelling physical and engineering systems with memory for hereditary effect and anomalous transport behaviour. The research presented here surveys and develops example applications of the Atangana-Baleanu operator in a spectrum of domains, including physics, engineering, signal processing, biology, epidemiology and financial mathematics. The fractional heat conduction and anomalous diffusion problems were formulated with the AB-Caputo derivative and solved using generalised Laplace transform; the engineering applications were evidenced using the fractional RC circuit and viscoelastic constitutive equations; the signal processing applications were illustrated using fractional filtering and edge-preserving image processing operators; the biological applications were shown via the fractional population growth model and the artificial neural network model; the applications in epidemiology were illustrated by means of the fractional-based SIR model; and the applications in finance were illustrated with the fractional Black-Scholes model and the use of fractional arithmetic on time series sampled from stock market dynamics. Each model includes the governing AB fractional differential equation and either an analytical or semi-analytical solution, along with representative numerical schemes (finite-difference, predictor-corrector and Laplace-transform based) and their stability and error characteristics. A detailed comparative analysis indicates the advantages of the Atangana-Baleanu operator compared to the classical Riemann-Liouville, Caputo and Caputo-Fabrizio operators in all the application areas studied. The analysis also shows that the continuous, non-singular Mittag-Leffler kernel of the Atangana-Baleanu operator provides stable, precise and physically meaningful representations of dynamical systems that display memory dependency.

Keywords: Atangana–Baleanu operators, memory-dependent systems, anomalous diffusion, fractional circuits, fractional epidemic models, fractional finance, numerical simulation.

1. INTRODUCTION

Numerous physical and engineering systems demonstrate a behaviour that is dependent on both their current state and their entire history of states. Viscoelastic material, diffusion in porous media, electrical circuits that utilise dielectric relaxation, biological populations, disease spread and financial markets have been observed to have hereditary or memory-based dynamics, which can only be partially described by classical differential equations that use integer-order derivatives (Mainardi, 2010; Magin, 2006). Fractional calculus, which uses operators that take into account all previous values of a function rather than just its instantaneous rate of change, provides an excellent mathematical framework for such systems (Podlubny, 1999).The Atangana–Baleanu (AB) derivative — which is derived from the non-singular, non-local Mittag-Leffler kernel — is particularly attractive for use in applied modelling because it avoids the kernel singularities that are present in classical derivatives (such as Riemann–Liouville and Caputo) and retains some qualitative long-range memory effects. In contrast, the Caputo-Fabrizio operator, which uses an exponential kernel that decays rapidly, does not have this memory retention effect (Atangana & Baleanu, 2016; Atangana & Gómez-Aguilar, 2018). Since its introduction, the AB operator has been used to solve many different types of physical and engineering problems, including heat transfer, groundwater flow, electrical circuit theory, epidemiology, and mathematical finance (Atangana, Gómez-Aguilar & Baleanu, 2018; Alkahtani, 2016).This paper reviews representative models of AB fractional systems and develops their governing equations and analytical/semi-analytical solutions across six major categories: physics; engineering systems; signal processing; biology; epidemiology; and finance. For each category, the governing equation is presented, along with analytical/semi-analytical solutions (if available), and a summary of the numerical methods used to verify the model's dynamics. The analytical framework (using extended definitions of fractional operators, Laplace transform, and Existence-uniqueness theory) that accompanies the theoretical paper "Analytical Properties and Solution Methods of Differential Equations Involving Atangana–Baleanu Fractional Operators" is applied to each model represented in this study.

2. MATHEMATICAL PRELIMINARIES

For a function $f \in H^1(a,b)$ and order $\alpha \in (0,1)$ the Atangana–Baleanu–Caputo (ABC) derivative is

$$D^{\alpha}_{ABC}\, f(t) \;=\; \frac{B(\alpha)}{1\,-\,\alpha}\int_{a}^{t} f'(\tau)\, E_{\alpha}([\,-\alpha(t-\tau)^{\alpha}\,/\,(1-\alpha)\,])\, d\tau$$

where $B(\alpha)$ is a normalization function with $B(0) \;=\; B(1) \;=\; 1$ and E_{α} is the one-parameter Mittag-Leffler function (Atangana & Baleanu, 2016). The corresponding Atangana–Baleanu integral is

$$I^{\alpha}_{AB}\, f(t) \;=\; \frac{1\,-\,\alpha}{B(\alpha)}\, f(t) \;+\; \frac{\alpha}{B(\alpha)\,\Gamma(\alpha)}\int_{a}^{t} (t\,-\,\tau)^{\alpha-1}\, f(\tau)\, d\tau$$

and the generalized Laplace transform pair

$$L\{\, D^{\alpha}_{ABC}\, f(t)\,\}(s) \;=\; \left[\frac{B(\alpha)\, s^{\alpha}}{(1-\alpha)s^{\alpha}\;+\;\alpha}\right][L\{f\}(s) \;-\; f(a)/s]$$

This document presents an analysis of the linearity of the models in the below. The definitions given and the properties they define are given in full generality with their corresponding conditions of existence, uniqueness, and stability developed in the accompanying theoretical paper by Kumar et al. (2018).

3. APPLICATIONS IN PHYSICS

Fractional operators have their most direct physical motivation in systems that are influenced by the history of the medium (e.g., by the medium's density).

Theoretical Foundation of Memory Effects in Physics

The physical memory effects occur when the flux or response at time t depends not only on the state of the medium at time t, but also on the history of the medium between times $[a\ t]$. The fractional operator is a natural encoding of this type of memory effect since it represents a nonlocal relationship between past fluxes and current responses. The Mittag-Leffler kernel associated with the AB derivative provides a way to interpolate smoothly between the behaviour of a system with short-term (near-Markovian) memory and the behaviour of a system with long-term (strongly nonlocal) memory as the parameter α varies in the interval $(0, 1)$, (Hilfer, 2000; Chen, Sun, Zhang & Korosak, 2010).

Anomalous Diffusion

The classical diffusion equation $\partial u/\partial t = D\,\partial^2 u/\partial x^2$ is generalized to the AB fractional anomalous diffusion equation

$$D^{\alpha}_{ABC}\,u(x,t) = D\,\frac{\partial^2 u}{\partial x^2},\ \ 0 < \alpha \leq 1,\ \ u(x,0) = u_0(x)$$

By using a generalized Laplace transform in t and a Fourier transform in x, the equation can then be expressed as an algebraic relationship for the transformed variable that, when reversed, gives us a spatial convolution of the initial condition with a Mittag-Leffler type Green's function. When $\alpha < 1$, the mean square displacement just grows at less than linear in time (sub-diffusively), thus reproducing the sub-diffusive behaviour present in porous media and biological transport and conforming to the anomalous-diffusion theories of Metzler and Klafter (2000) and the AB-specific considerations have been of Atangana and Gomez-Aguilar.(2018).

Fractional Heat Conduction

The original motivation of the fractional operator by Atangana and Baleanu can be characterized by the fractional heat-conducting equation,

$$D^{\alpha}_{ABC}\,T(x,t) = \kappa\,\frac{\partial^2 T}{\partial x^2}$$

Here T indicates the temperature being measured at the point of interest, κ represents the thermal conductivity or diffusivity associated with heat transfer throughout an extended body of material. When you apply the Laplace Transform method to solve this model for an infinite rod with some boundary temperature specified, you can generate the temperature profile in terms of the Mittag-Leffler Function. The Mittag-Leffler Function approximates the classical error function temperature solution for α containing a value near1. However, for α values further away from unity, we see a similar temperature profile but one that exhibits a flatter and more extended tail due to non-Fourier heat conduction associated with these types of materials that have memory (Gao, Yang & Baleanu, 2017).

4. APPLICATIONS IN ENGINEERING SYSTEMS

The real-world examples of fractional order memory models mainly occur in the applications of Electrical and Mechanical Engineering.

Fractional RC Circuit

The fractional order charging equation of a capacitor in a resistor-capacitor circuit with output from an applied voltage ($V(t)$) source is given by

$$D^{\alpha}_{ABC}\, q(t) + \frac{q(t)}{RC} = \frac{V(t)}{R}, \quad q(0) = q_0$$

where q is the charge on the capacitor. Using the linear-equation solution of the companion theoretical paper with $\lambda = 1/(RC)$, the charge response is

$$q(t) = q_0 E_{\alpha}(-\lambda * t^{\alpha}) + \int_0^t (t - \tau)^{\alpha-1} E_{\alpha,\alpha}(-\lambda * (t - \tau)^{\alpha})\, V(\tau)/R\, d\tau$$

While the classical exponential RC Discharge law is replaced by a Mittag-Leffler decay, the shape of the charging/discharging profile at an initial rate is close to that of the classical exponential curve, however; the long-term tail is much longer than that predicted by the integer-order RC model. Hence, the Mittag-Leffler decay has been successfully used to model both dielectric relaxation and supercapacitor behaviour more accurately than the integer-order RC model (Alkahtani, 2016; Ponce & Gómez-Aguilar, 2020).

Viscoelastic Systems

Fractional constitutive equations are often used to describe the viscoelastic stress-strain behaviour

$$\sigma(t) = E\, D^{\alpha}_{ABC}\, \varepsilon(t)$$

where σ is stress, ε is strain, and E is a generalized modulus. The AB-based model interpolates smoothly between purely elastic behaviour ($\alpha \to 0$) and purely viscous behaviour ($\alpha \to 1$), and since its kernel is non-singular it does not exhibit the numerical difficulties that are associated with the singular power-law kernel of the traditional fractional Kelvin-Voigt and Maxwell models, while it still reproduces the characteristic stress-relaxation and creep curves of both polymer and biological tissue (Carpinteri; Cornetti &Sapora, 2011; Rossikhin & Shitikova, 2010).

5. SIGNAL PROCESSING APPLICATIONS

Fractional-order operators offer additional design freedom in filter and image-processing applications by allowing a continuous order parameter that interpolates between the underlying integer-order operations.

Fractional Filtering

Fractional order operators provide more flexibility when designing filters and processing images because there is a continuous order of the filter that can be chosen between two integer type filtersby

$$D^{\alpha}_{ABC}\, y(t) \;+\; a\, y(t) \;=\; b\, x(t)$$

where x is the input signal, has a frequency response obtained from the Laplace transform by setting $s = j\omega$. The magnitude response will roll-off at a rate of α and this variable will allow for more flexible design options than traditional filters using integer order.

Image Processing

When using a fractional AB-Caputo spatial derivative instead of the traditional Laplacian operator in image restoration and performing edge detection, there is a parameter α to define the amount of smoothing (when small) versus the number of sharpening edges (closer to 1). The reason this new technique uses the Mittag-Leffler kernel is because it is non-singular so that when creating the fractional mask, you don't encounter the problems with truncation due to single weights like you would with power-law fractional masks. This has been a major reason for using this method in the field of texture-preserving denoising algorithms (Owolabi, 2019).

Biological Models

Adaptation, saturation and delayed response are characteristics of biological populations and neural networks; all these characteristics can be described with fractional-order memory operators.

- **Fractional Population Growth Model**

The classical logistic growth model is generalized to

$$D_{ABC}^{\alpha} N(t) = r N(t) (1 - N(t)/K), \; N(0) = N_0$$

The qualitative behaviour of the fractional-order operators, denoted by a fraction order α, can be described using the worked examples given in the companion theoretical paper. The fractional order α represents the degree to which the current rate of growth is influenced by the cumulative distribution of population size rather than the present instantaneous population size. Using numerical simulation, it was found that decreasing α leads to a decrease in the rate at which the population approaches the carrying capacity K. This is because populations subject to resource memory rely upon the cumulative distribution of resource use and have delayed density-dependent regulation (Rahimy, 2010).

- **Neural Network Models**

Fractional-order neuron models replace the classical leaky-integrator equation with

$$D_{ABC}^{\alpha} v(t) = -v(t)/\tau_m + I(t)/C_m$$

Where v is membrane potential, I is input current, and τ_m, C_m are membrane time and capacitance constants. The Mittag-Leffler kernel adds spike-frequency adaptation and history-dependent excitability directly to the dynamics of the membrane, allowing for a natural reproduction of many of the adaptation effects found in real-world cortical neurons that are not accounted for by classical exponential-decay models (Machado & Mata, 2015).

Epidemiological Models

Fractional-order epidemic models capture the influence of past infection history on current transmission dynamics.

- **Fractional SIR Model**

The classical Kermack–McKendrick SIR model is generalized to the AB fractional system

$$D_{ABC}^{\alpha} S(t) = -\beta S(t) I(t)$$

$$D_{ABC}^{\alpha} I(t) = \beta S(t) I(t) - \gamma I(t)$$

$$D_{ABC}^{\alpha}\, R(t) \;=\; \gamma\, I(t)$$

with $S(t) + I(t) + R(t)$ constant. The basic reproduction number for the fractional model is

$$R_0 \;=\; \beta\, S(0) \,/\, \gamma$$

The current SIR epidemic modeling framework is structurally the same as the classical SIR model but instead, utilizes the Mittag-Leffler kernel instead of an exponential kernel to determine the instantaneous approach to the peak of the epidemic as well as the post-peak tail recovery phase of the epidemic. The use of the Mittag-Leffler kernel allows for a slower decline in the epidemic curve, which fits the historical reports of the prolonged-tail effect that occurs after an outbreak has occurred. In a companion paper on the theoretical framework of fixed-point theory, Khan et al. (2020) have utilized a component-wise approach to evaluate the existence, uniqueness and stability of disease-free and endemic equilibria in the SIR epidemic modeling system.

6. FINANCIAL MODELLING APPLICATIONS

Long range dependent financial time series and the phenomenon of volatility clustering are motivations behind fractional-order extensions of classical market and option pricing models.

Fractional Black–Scholes Equation

The AB fractional Black–Scholes equation for option price $V(S, t)$ is

$$D_{ABC}^{\alpha}\, V \;+\; \frac{1}{2}\,\sigma^2\, S^2\, \frac{\partial^2 V}{\partial S^2} \;+\; rS\, \frac{\partial V}{\partial S} \;-\; rV \;=\; 0$$

Options with terminal payoffs at time T, for example. The generalized Laplace transform in t attempts to reduce the options pricing equation to a regular differential equation (ODE) in S for each transform variable s that can be solved and inverted numerically to give the option price surface; as a result of $\alpha \;<\; 1$, the option pricing equation diverges from the classical Black-Scholes compensation prices collected or derived from data and observations of both similarities and memory effects in the actual option markets (Mainardi, Gorenflo, and Scalas 2004).

Fractional Stock Market Dynamics

A fractional stochastic model for asset price $S(t)$ can be written as

$$D^{\alpha}_{ABC}\, S(t) = \mu\, S(t) + \sigma\, S(t)\, \xi(t)$$

The non-locally weighted Mittag-Leffler function based on previous price changes serves as a means of creating one way to explain the long-range autocorrelation observed in financial return data in a deterministic manner, and thus complements fractional Brownian motion that is purely stochastic in nature and has long memory. There is also a stochastic noise term $\xi(t)$ which is applied in conjunction with the Mittag-Leffler function to produce returns(Tarasov, 2019).

Numerical Simulation and Computational Validation

For the majority of the nonlinear AB fractional models described previously, it is not possible to obtain closed-form solutions and hence numerical schemes are used to simulate and validate them.

- **Finite Difference and Predictor–Corrector Schemes**

A very popular method of obtaining numerical solutions is by means of the use of an Adams-Bashforth type of quadrature, which discretizes the AB-Caputo derivative by using the Mittag-Leffler kernel, to construct a predictor-corrector time-stepping method

$$y_{n+1} = y_0 + \frac{1 - \alpha}{B(\alpha)} f(t_n, y_n) + \frac{\alpha\, h^{\alpha}}{B(\alpha)\, \Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{n,j}\, f(t_j, y_j)$$

where the weights $b_{\{n,j\}}$ arise from the discretized Mittag-Leffler kernel and h is the time step (Kai & Wang, 2020; Diethelm, Ford & Freed, 2002).

- **Stability and Error Analysis**

For the linear test equation

$$D^{\alpha}_{ABC}\, y = \lambda\, y$$

The predictor-corrector approach is inherently stable for $Re(\lambda)$ [less than] 0 if step sizes h are chosen so the discretized Mittag-Leffler weights stay bounded. Also, if f satisfies standard smoothness assumptions, then the local truncation error from this approach has the form $O(h^{(1+\alpha)})$, consistent with other fractional Adams-type methods reported as having comparable error orders. (Diethelm, Ford & Freed, 2004).

- **Representative Computational Results**

By applying the method to the fractional logistic model , the fractional SIR model, and the fractional RC circuit model for values of α in the range of {0.5, 0.7, 0.9, 1.0} confirms the predicted qualitative behaviours that were discussed earlier in each section: smaller values of α lead to a slower approach to equilibrium, longer epidemic tail, and slower relaxation of the RC circuit; the numerical solutions for α = 1 agree with the respective classical integer order solution to within the discretisation error, therefore providing a check on both the analytical formulation of this result and its numerical implementation (Owolabi, 2019; Momani & Odibat, 2008).

7. COMPARATIVE PERFORMANCE WITH CLASSICAL FRACTIONAL OPERATORS

In the above-discussed applications, the Atangana-Baleanu operator has three equivalent modelling advantages over classical fractional operators. First, its non-singular kernel eliminates any numerical stiffness associated with the singular power-law kernels of Riemann-Liouville and Caputo derivatives at the point where $\tau = t$, making the discretisation schemes of simpler, and increasing numerical stability at the same-step-size (Atangana et al., 2018). Second, the genuinely non-local, algebraically decaying Mittag-Leffler kernel better preserves long-range memory than the exponentially decaying kernel of Caputo-Fabrizio operator, and is thus reflected in the heavier-tailed relaxation, diffusion, and epidemic decline curves respectively, which correlate more closely with long-tail behaviour seen experimentally in many of the systems surveyed (Hilfer & Luchko, 2018). Finally, since the AB operator reduces to the classical Caputo derivative for α → 1, all the models generated in this study will converge to exactly corresponding classical integer-order models in this limit, providing a means to regard fractional order α as a single tuning parameter that can vary continuously between classical and strong-memorised dynamics, without having to change families of operators when the influence of memory is small (Sun et al. 2018).

8. CONCLUSION

This paper provides an extensive analysis of many fractional models of Atangana-Baleanu (AB) spanning a considerable number of disciplines such as physics (e.g. anomalous diffusion, heat conduction), engineering (e.g. RC circuits, viscoelastic systems), signal processing (e.g. filtering, image processing), biology (e.g. population growth, neural adaptation), epidemiology (the fractional SIR model) and finance (the fractional Black-Scholes model and stock market dynamics). For each of the models investigated, an AB fractional differential equation that represents the dynamics of the system was derived. Solutions to these equations (analytical or semi-analytical) were provided using both fixed-point methods and the generalised Laplace transform which were outlined in the theoretical companion of this work. Additionally, numerical approaches to solve the fractional systems using a finite difference scheme in a predictor/corrector fashion were developed along with discussion of the stability and error properties of the numerical techniques. Based upon a comparative discussion of all of the models, it was concluded that the Atangana-Baleanu fractional operators, as defined using the genuinely nonlocal and non-singular Mittag-Leffler kernel, provide a unique and consistent advantage over the Riemann-Liouville, Caputo and Caputo-Fabrizio fractional operators across all six application areas while also demonstrating that classical integer order behaviour can be continuously approached as α approaches unity. These conclusions allow for the continued utilisation of the Atangana-Baleanu fractional operator as a physically realistic and flexible framework to describe memory-dependent types of physical and engineering systems, while also proposing several areas of needed future work such as developing multi-dimensional AB fractional partial differential equation models, stochastic AB-fractional systems and developing a data driven approach to estimating the fractional order α from experimental time series.

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